HAZARD (INFANCY) ANALYSIS SPECIFICATION

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ABSTRACT: This document outlines hazard test theory, explains decision rules for each test, and interprets the statistical decisions by detailing the procedure for analyzing a product's relationship between hazard rate and operating time. Product infancy and steady-state failure rates are estimated after the user answers a series of questions using the HAZFIT software.

APPLICABILITY: Recommended for use by reliability and manufacturing engineers and data analysts measuring failure rate as a function of operating time or when a hazard function estimation is required. Hazard function measuring is done during process maturity testing, infancy analysis, burn-in analysis, and during wearout analysis of electromechanical options and components.

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1 INTRODUCTION

1.1 SCOPE

This document describes an automated statistical tool used to analyze a product's hazard data. The tool chosen is not the only one available, however, it has been recognized as the most successful analysis method within Digital to date. Referenced Documents a. through d. provide a more complete and theoretical treatment of the data analysis problem.

Information is provided for the user to access the computer program HAZFIT that performs product hazard estimation and analysis. The appendix provides detailed examples of model input, user interface, and output.

To obtain assistance in fitting a hazard curve from data, call DTN: 223-6845.

1.2 GENERAL THEORY

Hazard estimation is a procedure used to determine the functional relationship of failures over a time period. The hazard estimate is performed over a class of curves that share a common rapid decrease in failure rate. Each hazard curve is described by three parameters that enable it to take on a variety of shapes and sizes. The hazard function h(t) is determined using the following equation.

$$h(t) = LAMBDA + ALPHA\, e^{-(BETA * t)}$$

where:

t = operating time in hours

LAMBDA = steady-state failure rate approached as t increases

ALPHA = infancy portion of the hazard rate that may be removed by product burn-in

BETA = infancy decay rate indicating how quickly the hazard decreases

NOTE

h, t, LAMBDA, ALPHA, and BETA are positive numbers.

The estimated hazard curve that explains the observed hazard data has its parameters LAMBDA, ALPHA, BETA estimated through a method called maximum likelihood estimation. This technique tries to provide the BEST explanation of the observed data set, based on observing the data set under the specified conditions.

The LAMBDA, ALPHA, and BETA maximum likelihood estimates (MLE) are determined using the numerical technique Newton-Raphson iteration (see Referenced Document d). Estimate improvements are achieved by repeated iteration. The MLE for the hazard curve fit is the point where the iteration values converge. No simple formulas are available for the parameter estimates.

1.3 DEFINITIONS

Elbow - The point or interval of time when the hazard function begins to level off.

Failure Rate - The average number of failures per time unit, equal to the reciprocal of the MTBF.

Fisher Information Matrix - The matrix containing joint information regarding the accuracy of the true hazard parameter estimation.

Hazard Function - The instantaneous failure rate defined at time t as the rate of change of the conditional probability of failure, given the unit survives up to time t.

Infancy Decay Rate - The rate at which the hazard function decreases to steady state.

Likelihood Function - The probability of observing actual test data: a function of fixed hazard parameter values.

Maximum Likelihood Estimates (MLE) - Estimates for the hazard function parameters found by solving the normal equations, thus maximizing the likelihood function.

Mean Time Between Failure (MTBF) - The expected operating time between first use of a machine (or end of repair) and the next observed failure.

Newton-Raphson Iteration - A mathematical technique using local quadratic approximations of the likelihood function to find the maximum and obtain the hazard parameter MLE values.

Poisson Distribution - A distribution describing the statistical behavior of the number of failures for a wide range of equipment.

Reliability Function - The survival function defined at time t as the probability of survival to time t.

Significance Level - The probability of rejecting the hypothesis when it is true.

Steady-state Failure Rate - The rate of machine failure attained after infancy is completely removed.

Steady-state MTBF - The MTBF attained after infancy removal.

Time To Burn-in - The time necessary to operate a newly manufactured product to meet a desired level of reliability.

Total Hazard - The expected number of machine failures during an interval of time (equal to the area under the hazard curve during the time interval).

2 DERIVATION OF HAZARD ESTIMATE

2.1 ISSUES

Hazard curve fitting is used for burn-in analysis of manufactured products. Hazard data analysis is performed by HAZFIT through the following questions and the answers provided through the use of statistical tests where appropriate.

Question 1. What form of data is required?

Question 2. How much is enough testing?

Question 3. Is there infancy present?

Question 4. Can the steady-state MTBF be measured?

Question 5. What hazard function best fits the data?

Question 6. Does the fitted curve explain the infancy?

Question 7. Is the failure specification ever to be met?

Question 8. If the failure specification is to be met, how much burn-in is needed?

Question 9. How much burn-in is required to avoid infant failures?

2.2 ASSUMPTIONS

The HAZFIT analysis depends on the following unrestrictive assumptions. They are general and reasonable for use, and mathematical arguments can be made to support these assumptions.

The assumptions are:

a. Failure counts for the entire test bed over non-overlapping (disjoint) time intervals are independent of each other.

b. The number of failures for each machine over a specified time interval is a distributed Poisson with the mean equal to the total hazard over the interval.

These assumptions define the properties of a non-homogeneous Poisson process.

2.3 PROCEDURE

HAZFIT is an interactive computer model. During a HAZFIT interactive user session, each question, detailed below, is asked and then statistically tested. The analysis method is provided, but the estimate detail and actual analysis are not. Below are the hypotheses, test statistics, and decision rules.

Referenced Document b. describes large sample and small sample implications, formula for parameter estimates, and variance considerations of the estimation process.

Statistical tests are used to answer the ten analysis questions. A gray-area response is valid when a test statistic exceeds the critical value corresponding to the 15% significance level. A firm response is valid when a test statistic exceeds the critical value at the 5% level. The percentage levels are arbitrary, but acceptable and recommended.

2.3.1 Question 1: What Form of Data is Required?

The time axis consists of intervals of increasing length as failures are tallied. Two data items are required for each time interval: the number of failures, and the accumulated time for the test bed operation. These numbers are as follows:

```
< N1, N2, . . ., Nk >     Number of failures
< M1, M2, . . ., Mk >     Machine operating time
```

Examples of time partitions (in hours) are:

```
< 1, 2, 4, 6, 10, 14, 20, 40, 70, 100, 150, 200 >
< 1, 2, 5, 10, 20, 30, 50, 75, 100, 150, 200 >
```

When fewer failures occur and grosser tallies are justified use:

```
< 2, 10, 20, 40, 70, 100 >
< 1, 4, 10, 25, 50, 75, 100 >
```

NOTE

Other intervals may be chosen depending on the desired test duration or availability of data. In particular, HAZFIT may be used to analyze field data broken down by month.

It is best to have the time axis broken up so that most of the intervals contain at least one failure. The more total failures occurring, the better quality the resulting hazard fit. No requirement is imposed on zero-failure interval data. However, zero counts cannot be tolerated by the algorithm for time intervals early in the test. Therefore, zero counts should be avoided, if possible.

Early interval grouping improves the possibility that the algorithm will find significant hazard estimates. However, early interval grouping will give conservative hazard-curve measurement, conservative estimates of the infancy decay, and conservative steady-state MTBF. Conservative analysis results from diluted hazard data. Accurate estimates are required for a meaningful hazard curve fit. Therefore, data grouping should be minimized. Initial, limited data from new process lines will be of limited use in developing an accurate hazard curve. Small partitions with multiple failures will yield the best results.

2.3.2 Question 2: How Much is Enough Testing?

A good answer to this question depends on the sufficiency objective. Enough testing to accomplish what, to get a good hazard curve fitted from the data, to make good decisions regarding burn-in, or to feel good about burn-in analysis? To fill most needs, a satisfactory answer must accommodate the following requirements:

a. The total operating time should be at least twice the MTBF goal and preferably four to five times that goal.

b. In order to measure infancy decay rate, sufficient failures must occur early in testing.

c. In order to measure a steady-state MTBF, sufficient failures must occur late in testing when machine reliability has stabilized. Some questions cannot be answered without late failures occurring. In particular, the steady state estimate will not be produced.

d. Many overall failures must occur and some failures should occur in each cell. Decision rules with inaccurate critical values result when too few failures occur or when the ratio of the highest to lowest expected failures is much bigger than two or three (that is, the expected number of failures in each partition should be about the same).

2.3.3 Question 3: Is There Infancy Present?

To test whether or not there is infancy present, the constant failure rate hypothesis is tested. The chi-square goodness of fit test, conditional upon the total number of failures observed in the test bed, is the test used. Large test statistic values indicate a lack of agreement between the interval failure counts and what should be observed when the hypothesis is true. If the hazard rate is constant over time, the number of failures in each time interval should be proportional to the test time accumulated over that interval by the entire test bed of machines.

The null hypothesis to be formally tested is:

```
H: constant failure rate
```

which implies that expected failure counts are proportional to the operating time accumulated during the corresponding intervals. The expected failure counts are denoted:

```
< E1, E2, . . ., Ek >
```

are such that Ej = TOTF (Mj / TOTM) for j=1 to k

```
where:      Ej = expected number of failures in j-th interval
            Mj = accumulated operating time in j-th interval
            k  = number of time intervals
          TOTF = total number of failures
          TOTM = total machine operating time.
```

The test statistic is conditioned upon TOTF taking place. The goodness of fit statistic (Q1) is a large value when a large discrepancy exists between the observed failure counts and the counts expected under the null hypothesis. Q1 is a small value when no discrepancy is evident in the data. The test statistic is

$$Q1 = \frac{(N1-E)^2}{E} + \frac{(N2-E2)^2}{E2} + \ldots + \frac{(Nk-Ek)^2}{Nk}$$

where k equals the overall number of time intervals.

The three decision rules are:

a. Reject H if Q1 > X2 (.05; k-1)

b. Consider rejecting H if Q1 > X2 (.15; k-1)

but Q1 < X2 (.05; k-1).

c. Otherwise accept H where the chi-square cutoffs at given levels and degrees of freedom serve as critical points.

Rejection of the constant hazard hypothesis leads to three possible alternatives; infancy, wearout, or erratic data. Significant infancy is common in PMT. Wearout should never be seen in PMT. The HAZFIT model will not produce good hazard function estimates for the wearout situation. Erratic data can come about from insufficient information or a host of other reasons. If you believe you have insufficient data, contact your analyst.

Infancy exists when the ratio of the simple MTBF estimate before and the simple MTBF estimate after the elbow significantly exceeds one. Wearout exists when the ratio is significantly smaller than one. Problems may arise in the following questions (4 through 9) if a significant amount of infancy is not detected at this stage.

If H is accepted, there is insufficient evidence of infancy and the analysis may be concluded with a constant failure rate assumed.

If you receive "WARNING–POSSIBLE DIFFICULTY IN HAZARD ESTIMATION!!!" and you continue with the HAZFIT analysis, the accuracy of the results should be considered suspect.

2.3.4 Question 4: Can the Steady-state MTBF be Measured?

In order to test the hypothesis that the steady-state parameter LAMBDA does not belong in the true hazard function, the likelihood ratio test is used. Under the null hypothesis

```
H: LAMBDA = 0
```

the test statistic Q2 is distributed chi-square with one degree of freedom, where:

$$Q2 = -2 \ln \frac{L(0, ALPHA, BETA)}{L(LAMBDA, ALPHA, BETA)}$$

and L is the likelihood function of the data (see Referenced Document b).

The decision rule is:

```
reject H  when  Q2 > X2 (.05; 1) = 3.841

accept H  otherwise.
```

There is no opportunity to make a gray-area decision since one of two models must be chosen. A rejection means a "YES" answer; an acceptance means a "NO" answer.

The HAZFIT algorithm estimates both models. If the additional parameter LAMBDA gains a significant increase in the likelihood function, the program settles on the full model and makes inferences regarding the steady state. If not, the reduced model is used and nothing can be inferred about the eventual reliability.

Beware: If you have been told that a hazard fit may be difficult, this decision may be wrong.

2.3.5 Question 5: What Hazard Function Best Fits the Data?

When the maximum likelihood estimate (MLE) of the relevant parameters are known, the unknown true values are replaced by their estimates in the hazard function equation. The hazard fit is defined as the resulting function when the MLE is substituted for LAMBDA, ALPHA, and BETA in the following formula for h(t).

$$h(t) = LAMBDA + ALPHA\ e^{-(BETA * t)}$$

Question 4 determined the relevant model to use in maximum likelihood calculations. LAMBDA equals zero in the two parameter model.

2.3.6 Question 6: Does the Fitted Curve Explain the Infancy?

This formalizes the question of whether the fitted hazard curve is adequate for use. The statistical test consists of testing the constant failure hypothesis versus the alternative model chosen in question 5. The likelihood ratio test is used again. Large values of the test statistic indicate that the alternative hypothesis is likely to be true rather than the constant hypothesis. The test statistic may be small when a decreasing curve is only vaguely apparent from the data.

Under the null hypothesis

```
H: constant failure rate
```

the test statistic Q3 is distributed chi-square with one degree of freedom when the reduced model is chosen and equals two when the full model is chosen. Define

$$Q3 = -2 \ln \frac{L(\text{constant model})}{L(\text{chosen model})}$$

and L is the likelihood function of the data. See Referenced Document b for further details.

The decision rule is:

a. Reject H when Q3 > X2 (.05; p-1).

b. Consider rejecting H when Q3 > X2 (.15; p-1) but Q3 < X2 (.05; p-1).

c. Otherwise accept H

where p hazard parameters appear in the chosen model.

2.3.7 Question 7: Is the Failure Specification Ever to be Met?

Test the hypothesis that the failure specification is eventually met. The test statistic is the normalized MLE estimate for the steady-state failure rate LAMBDA. The normality approximation according to the classical maximum likelihood theory is used here. Large values of the test statistic indicate that the estimated steady-state rate is many standard errors greater than the failure specification. The approximation for the test statistic distribution has the greatest accuracy when a large total number of failures is observed.

Under the null hypothesis

```
H: LAMBDA <= failure specification
```

the test statistic Z has approximate normal distribution with a mean of zero and a standard error of one, where:

$$Z = \frac{\text{LAMBDA MLE} - \text{failure spec}}{\text{standard error (LAMBDA)}}$$

and the estimated standard error is available from the Fisher Information matrix. See Referenced Document b. for details.

The decision rule is:

a. Reject H when Z > z(.15).

b. Consider accepting H when Z > 0

but Z < z(.15) = 1.036.

c. Otherwise accept H.

A rejection means a "NO" answer to the question. Considering acceptance means that the manufacturing process is close to meeting the specification. An acceptance means a "YES" answer.

2.3.8 Question 8: If the Failure Specification is to be Met, How Much Burn-in is Needed?

The time to burn-in new products so that they will at least meet the specified reliability is the time necessary to experience a decrease in the hazard curve to the specification level. To obtain the solution, set the fitted hazard function to the specification level and solve the equation. It is simple if the specification exceeds the fitted steady-state failure rate LAMBDA.

The time to burn-in can be estimated by the inverse of the hazard function. It is

$$h^{-1}(\text{spec}) = \text{BETA}^{-1} \ln \frac{\text{ALPHA}}{(\text{spec} - \text{LAMBDA})}$$

where the parameters LAMBDA, ALPHA, and BETA are replaced by their estimates. Set LAMBDA to zero in the reduced model.

2.3.9 Question 9: How Much Burn-in is Required to Avoid Infant Failures?

The expected number of infant failures is proportional to the total hazard above the steady-state rate; the constant being the volume of the product. The hazard described is the area under the hazard curve, but above the steady-state rate, over the entire time axis. This area is always finite according to the type of model chosen. As time passes, the proportion of that area to the total area approaches one. When a given proportion of the total hazard has exceeded the steady state the proportion of infancy has been removed. Infancy has been removed since the expected infant failures have been avoided. The solution depends upon the decay rate BETA. See Referenced Document b for more details.

HAZFIT provides answers for various proportions of infancy removal. They are:

< .75, .90, .95, .96, .97, .98, .99, .995, .999 >

The MTBF level associated with such burn-in times is also reported.

CAUTION

If the steady state MTBF cannot be measured, hazard estimates beyond the range of the actual data are likely to be highly inaccurate.

3 SENSITIVITY ANALYSIS: HOW SENSITIVE ARE THE RESULTS WHEN DATA IS SLIGHTLY DIFFERENT?

Several contingencies may be investigated in order to judge the sensitivity of the answers to some or all of the above questions. Changes in the answers after considering different hazard data sets arise from hypothetical situations. Sensitivity of answers may be addressed to achieve comfort with the analysis. Common examples are:

a. Test machines ran failure-free for an additional 24 hours.

b. Twice the total test hours produced twice the number of failures in each time interval.

c. Twice the number of failures occurred in the last three time intervals in twice the accumulated operating time over those intervals.

d. Some of the failures that occurred in the first hour were labeled dead on arrival (DOA) and were deleted from the data set unless too numerous.

e. Designated failures are to be deleted since they are all attributed to software problems.

f. Failures pertaining to a particular machine are to be ignored because the problems are too numerous and therefore the unit is not representative.

APPENDIX A

ACCESS TO AND AN EXAMPLE OF HAZFIT

This appendix provides an example of how the model HAZFIT has been used. It provides accessing instructions, operating instructions, and the report output.

The Data:

This example uses real burn-in data from a manufacturing plant. Prior to using the model, the data should be tabulated as it is below, by time interval, number of failures per interval, and operating time per interval.

Table 1: Sample Data

Time Interval (hours)	Number of Failures	Operating Time (hours)
0.0 - 0.5	30	400
0.5 - 1.0	52	380
1.0 - 1.5	36	358
1.5 - 2.0	14	345
2.0 - 2.5	3	341
2.5 - 3.0	8	338
3.0 - 3.5	5	335
3.5 - 4.0	6	332

Accessing HAZFIT: Local Access on a VAX/VMS System

You must have an account on your local VAX System with a minimum of 1000 blocks. This can be arranged through your system manager or whatever operational procedure is used in your facility. Also, ensure you have the capability to reach other nodes on the network.

Once you log into your VAX account, issue the following command:

```
$COPY CSSE::DISK$A6:[CSSEINFO.RAMP]HAZARD.BLD
```

The system will respond with:

```
$-To:
```

you respond: *.*

Next execute the following command:

```
$@HAZARD.BLD
```

This loads the executable copies of the HAZFIT programs into your area. This will probably take at least 30 minutes or longer, depending on the state of the network linking to your system.

When this process ends, it will return you to the monitor with a $ prompt.

At this point you are ready for a VAX HAZARD session. To do this, enter the following command:

```
RUN HAZARD
```

After reading that which the screen will present, enter:

```
RUN HAZFIT
```

Plotting has not been available on previous VAX versions of HAZFIT. This capability is now available. In order to obtain the plot, complete the following:

1. Run HAZFIT
2. Exit completely from HAZFIT to the point where another system command can be entered.
3. Enter the following command:

   @HAZPLT

The VAX version of HAZFIT mimics the TOPS-20 APL version. Data are input in exactly the same way for either version.

CAUTION

A prerequisite to using the "optional plot" in the VAX version is availability of the APL language interpreter on your VAX. Contact your system manager to determine if VAX-APL is available at your site. If you have an old version of HAZFIT and desire the capability to produce plots, copy the program over again from COIN::.

An example is shown on the following pages.

```
                    *********  QUESTION 1  *********

                     WHAT FORM OF DATA IS REQUIRED?

FOR SEVERAL INTERVALS OF OPERATING TIME, THE FAILURE COUNTS, THE
ACCUMULATED TEST TIMES, AND THE INTERVAL RIGHT ENDPOINTS ARE REQUIRED
BY THE PROGRAM IN ORDER TO FIT A HAZARD CURVE.

THE TIME AXIS MUST BE PARTITIONED (OMIT ZERO)
ENTER NAME OF VECTOR, OR RIGHT ENDPOINTS FOR THE TIME INTERVALS.
0.5  1  1.5  2  2.5  3  3.5  4

ENTER NAME OF VECTOR, OR ACTUAL INTERVAL FAILURE COUNTS.
30  52  36  14  3  8  5  6

HAVE ACCUMULATED OPERATING TIMES PER INTERVAL BEEN PREPARED?
     ENTER <0>  NO  <1>  YES

.BX:

  1

ENTER NAME OF VECTOR, OR ACTUAL INTERVAL ACCUMULATED OPERATING TIMES.
400  379.5  357.5  345  340.75  338.  334.75  332


SUMMARY OF THE DATA ENTERED FOR HAZARD ANALYSIS

         TIME       OBSERVED       MACHINE          OBSERVED
       INTERVAL      FAILURES      OPERATING TIME     FAILURE RATE

     0.0      0.5     30          400.0         0.075000000
     0.5      1.0     52         379.5         0.137022398
     1.0      1.5     36         357.5         0.100699301
     1.5      2.0     14         345.0         0.040579710
     2.0      2.5      3         340.8         0.008804109
     2.5      3.0      8         338.0         0.023668639
     3.0      3.5      5         334.8         0.014936520
     3.5      4.0      6         332.0         0.018072289

HAS THE DATA BEEN PROPERLY INPUTTED?        ENTER <0>  NO   <1>  YES

.BX:

  1

FOLLOWING THE ANSWER TO EACH QUESTION WILL BE A PAUSE

TO CONTINUE, SIMPLY HIT THE CARRIAGE RETURN  <CR>
```

```
                    *********  QUESTION 2  *********

                      HOW MUCH IS ENOUGH TESTING?

AVERAGE FAILURE RATE IS    0.05446507515
SIMPLE M T B F ESTIMATE IS   18.36038961

M T B F  SPECIFICATION IS   2000

TO MEASURE FAILURE RATE IN A GENERAL SENSE, THE TOTAL OPERATING
TIME SHOULD BE AT LEAST TWO TIMES THE MTBF GOAL, FOR TO FIVE TIMES
     THE GOAL IS VERY DESIRABLE.

TO MEASURE THE INFANCY DECAY RATE, SEVERAL FAILURES MUST OCCUR EARLY
     IN TESTING, ENOUGH TO MAKE THE INFANCY APPARENT.

TO MEASURE THE STEADY-STATE MTBF, AT LEAST A FEW FAILURES MUST OCCUR
     LATE IN TESTING WHEN THE RELIABILITY BECOMES STEADY.

TO SET THE CRITICAL VALUES IN THE DECISION RULES ACCURATELY, MANY
     FAILURES MUST OCCUR OVERALL (AT LEAST 20 OR 30).

     PAUSE    --    HIT <CR> TO CONTINUE

                    *********  QUESTION 3  *********

                       IS THERE INFANCY PRESENT?

TOTAL NUMBER OF FAILURES CONDITIONED UPON IS   154

HYPOTHESIS:  CONSTANT FAILURE RATE

CONDITIONAL CHI-SQUARE STATISTIC  Q1 = 102.4456799

     DOF = 7

CRITICAL VALUES FOR TEST STATISTIC Q1

          LEVEL       .15    .05

          CUTOFF     10.727  14.047

REJECT THE HYPOTHESIS WHEN Q1 > CUTOFF
```

****** DECISION ******

REJECT HYPOTHESIS

**** IN SOME CASES THE SIGNIFICANT DEPARTURE FROM CONSTANCY SHOULD PROPERLY BE IDENTIFIED AS ERRATIC DATA RATHER THAN EVIDENCE OF INFANCY WHEN THE FAILURE RATE IS NOT HIGH EARLY IN TIME AND LOWER LATE IN TIME.

NUMBER OF FAILURES UP TO 0.5 HOURS IS 30
NUMBER OF FAILURES AFTER 0.5 HOURS IS 124

AVERAGE TIME TO FAILURE UP TO 0.5 HOURS IS 13.33333333
AVERAGE TIME TO FAILURE AFTER 0.5 HOURS IS 19.5766129

****** CONCLUSION ******

DEPARTURE IS IDENTIFIED AS INFANCY

EXPECTED NUMBER OF FAILURES ARE BASED UPON THE CONSTANT FAILURE RATE ASSUMPTION STATED IN THE HYPOTHESIS.

TIME INTERVALS		OBSERVED FAILURES	EXPECTED FAILURES
0.0	0.5	30	21.7860
0.5	1.0	52	20.6695
1.0	1.5	36	19.4713
1.5	2.0	14	18.7905
2.0	2.5	3	18.5590
2.5	3.0	8	18.4092
3.0	3.5	5	18.2322
3.5	4.0	6	18.0824

PAUSE -- HIT <CR> TO CONTINUE

A MAXIMUM LIKELIHOOD CALCULATION IS TO BE MADE IN ORDER TO FIT THE HAZARD CURVE FOR THE GIVEN DATA. THE LIKELIHOOD FUNCTION IS MAXIMIZED WITH RESPECT TO THREE PARAMETERS -- LAMBDA, ALPHA, BETA.

ITERATION IS CONDUCTED VIA NEWTON-RAPHSON IN ORDER TO ARRIVE AT THE ESTIMATED PARAMETER VALUES TO USE IN THE FITTED HAZARD CURVE FOR BOTH THE 2-PARAMETER AND THE 3-PARAMETER MODELS. THE APPROPRIATE MODEL IS CHOSEN BASED UPON EXAMINATION OF THE IMPROVEMENT IN THE LOG LIKELIHOOD FUNCTION WHEN FITTING THE FULL MODEL.

HAZARD PARAMETER ESTIMATES ARE OBTAINED UNDER THE ASSUMPTION THAT FAILURES TAKE PLACE ACCORDING TO A NON-HOMOGENEOUS POISSON PROCESS.

I E INDEPENDENT FAILURE COUNTS ACROSS TIME INTERVALS AND
POISSON NUMBER OF FAILURES PER INTERVAL

HAZARD ESTIMATION PHASE PROCEEDS

THE 2 - PARAMETER HAZARD MODEL IS ESTIMATED

CRITERION FOR TERMINATION IS SATISFIED. ITERATION HAS PRODUCED A GOOD HAZARD FIT.

STEADY-STATE PARAMETER LB IS SET TO ZERO

FINAL ESTIMATES FOR THE 2-PARAMETER MODEL

LB = 0 ALPHA = 0.142479874 BETA = 0.6267824904

PARAMETER	ESTIMATE	STANDARD ERROR
ALPHA	0.142479874032	0.018041110261
BETA	0.626782490	0.082876037

EXPECTED NUMBER OF FAILURES IN THE FOLLOWING TABLE IS BASED UPON THE 2-PARAMETER HAZARD MODEL.

TIME INTERVAL		OBSERVED FAILURES	EXPECTED FAILURES
0.0	0.5	30	48.9257
0.5	1.0	52	33.9301
1.0	1.5	36	23.3639
1.5	2.0	14	16.4810
2.0	2.5	3	11.8986
2.5	3.0	8	8.6273
3.0	3.5	5	6.2456
3.5	4.0	6	4.5278

```
PAUSE  -- HIT <CR> TO CONTINUE
THE 3 - PARAMETER HAZARD MODEL IS ESTIMATED


FINAL ESTIMATES FOR THE 3-PARAMETER MODEL
     LB = 1.918835647-12    ALPHA = 0.142479874    BETA = 0.6261557079

EXPECTED NUMBER OF FAILURES IN THE FOLLOWING TABLES IS BASED UPON THE
     3-PARAMETER HAZARD MODEL

          TIME          OBSERVED        EXPECTED
       INTERVAL         FAILURES        FAILURES

     0.0       0.5         30          48.9257
     0.5       1.0         52          33.9301
     1.0       1.5         36          23.3639
     1.5       2.0         14          16.4810
     2.0       2.5          3          11.8986
     2.5       3.0          8           8.6273
     3.0       3.5          5           6.2456
     3.5       4.0          6           4.5278
```

```
                    *********   QUESTION 4   *********

              CAN THE STEADY-STATE  M T B F  BE MEASURED ?

HYPOTHESIS:   LAMBDA = 0  (NO DETECTABLE STEADY STATE)

CHI-SQUARE STATISTIC FOR INCLUSION OF STEADY STATE PARAMETER

     Q2 = 0
     DOF = 1

     CRITICAL VALUE FOR THE TEST STATISTIC AT THE 10 % LEVEL
              CUTOFF = 2.705543014

REJECT THE HYPOTHESIS WHEN Q2 > CUTOFF

                      ******   DECISION  ******

ACCEPT HYPOTHESIS  --  STEADY STATE CANNOT BE MEASURED

     PAUSE  --  HIT <CR>  TO CONTINUE

                  *********   QUESTION 5   *********

              WHAT HAZARD FUNCTION BEST FITS THE DATA ?

THE BEST FITTED HAZARD MODEL HAS THE FOLLOWING PARAMETERS PLOT OF
HAZARD DATA VERSUS THE FITTED HAZARD APPEARS AT END OF THIS OUTPUT

                      ******   DECISION   ******

     LB = 0         ALPHA = 0.142479874         BETA = 0.6267824904

HAZARD FUNCTION:  H(T) = LB + ALPHA $ EXP( -BETA $ T)

     PAUSE  --  HIT  <CR>  TO CONTINUE
```

```
                *********   QUESTION 6   *********

            DOES THE FITTED CURVE EXPLAIN THE INFANCY ?

HYPOTHESIS:  ZERO INFANCY PARAMETERS  (NO EXPLANATION)

LIKELIHOOD RATIO STATISTIC  Q3 = 67.41566306

     DOF = 1

CRITICAL VALUES OF TEST STATISTIC Q3

     LEVEL          .15        .05

     CUTOFF        2.072            3.841

REJECT HYPOTHESIS WHEN Q3 > CUTOFF

                    ******   DECISION   ******

           REJECT HYPOTHESIS  --  FIT EXPLAINS THE INFANCY

              TIME                OBSERVED          HAZARD
            INTERVAL                RATE              FIT

           0.0      0.5      0.075000000     0.122314310
           0.5      1.0      0.137022398     0.089407341
           1.0      1.5      0.100699301     0.065353536
           1.5      2.0      0.040579710     0.047771074
           2.0      2.5      0.008804109     0.034918929
           2.5      3.0      0.023668639     0.025524475
           3.0      3.5      0.014936520     0.018657469
           3.5      4.0      0.018072289     0.013637936

           PAUSE  --  HIT <CR> TO CONTINUE

                    *********   QUESTION 7   *********

              IS THE FAILURE SPECIFICATION EVER TO BE MET ?

        NO COMPARISON OF STEADY STATE TO M T B F SPEC CAN BE MADE
             SINCE THE STEADY STATE FAILURE RATE CANNOT BE MEASURED

             PAUSE  --  HIT <CR> TO CONTINUE
```

```
                *********   QUESTION 8   *********

IF THE FAILURE SPEC IS TO BE MET, HOW MUCH BURN-IN IS NEEDED ?

     M T B F          TIME TO BURN-IN
    SPECIFIED             (HOURS)

       2000               9.0180

     PAUSE  -- HIT <CR> TO CONTINUE

                *********   QUESTION 9   *********

   HOW MUCH BURN-IN IS REQUIRED TO AVOID INFANT FAILURES ?

PROPORTIONS OF EXPECTED INFANT FAILURES TO BE AVERTED
     REQUIRE A CERTAIN BURN-IN TIME FOR EACH MACHINE
     AND HAVE ASSOCIATED WITH THEM VARIOUS MTBF LEVELS

APPROPRIATE PROPORTIONS FOR USE ARE  .95  OR  .99 DEPENDING
UPON   WHETHER THE DETECTED INFANCY IS TYPICAL OR ENORMOUS.

       PROPORTION        TIME           M T B F

         0.750          2.212            28.074
         0.900          3.674            70.185
         0.950          4.780           140.371
         0.960          5.136           175.463
         0.970          5.595           233.951
         0.980          6.241           350.927
         0.990          7.347           701.854
         0.995          8.453          1403.707
         0.999         11.021          7018.535
```

```
        FIX PAPER TO TOP FOR THE PLOT  -- HIT <CR> TO CONTINUE

        PLOT OF FITTED HAZARD (+) AND OBSERVED HAZARD (*) VERSUS
        MIDPOINTS

0.150    +
         +
         +
         +        *
         +
0.125    +
         +  +
         +
         +
         +
0.100    +           *
         +
         +        +
         +
         +
0.075    +  *
         +
         +            +
         +
         +
0.050    +            +
         +
         +            *
         +                       +
         +
0.025    +                            +
         +                          +    *
         +                          *    +
         +                 *
         +
0.000    +    +    + +    +   +    +    +  +
         0        1      2        3      4

DO YOU WISH TO DELETE THE FIRST PERIOD FOR INCREASED RESOLUTION ?

   ENTER <0>  NO   <1>  YES
.BX:
     0
     )OFF
TTY11)  11:26:28 29-NOV-83
CONNECTED    0:17:05  CPU TIME  0:00:37
18212  STATEMENTS 73048  OPERATIONS
Killed Job 24, User RAMP .MODELS, Account 71F.77E00000, TTY  11,
  at 29-Nov-83 11:26:28,   Used 0:00:37 in 0:17:18

SWITCH A
HOST NAME:
```

A.1 DIGITAL TECHNICAL MEMOS

a. *Pros and Cons in Hazard Fitting*, memo by Jim Gale, RAMP Analysis at Digital, February, 1983

b. *Burn-in Estimation*, memo by Jim Gale, RAMP Analysis at Digital, September, 1983

Contact the memo author or the RAMP Analysis Group for copies.

A.2 BIBLIOGRAPHY

a. *Survival Models and Data Analysis*, by R. Elandt-Johnson & N. Johnson. Published by John Wiley & Sons, 1980

b. *Numerical Methods*, by G. Dahlquist. Published by A. Bjorck & N. Andersen, Prentice-Hall, 1974

The local Digital Branch Library will locate these references.

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